Experiment 10
Moment of Inertia

Advanced Reading

University Physics Volume 1 (OpenStax) text Sections 10.4, 10.5 and 10.7

Equipment

- •Beck's Inertia Thing (rotational apparatus)
- Vernier caliper
- masses
- meter stick
- stopwatch
- 50 g mass hanger

Objective

The objective of this experiment is to dynamically measure the moment of inertia of a rotating system and to compare this to a predicted value.

Theory

The moment of inertia can be viewed as the rotational analog of *mass.* Torque and angular acceleration are the rotational analogs of *force* and *acceleration*, respectively. Thus, in rotational dynamics, Newton's second Law (F=ma) becomes $\tau = I\alpha$, where τ is the (net) applied torque, I is the moment of inertia of the body and α is the angular acceleration.

An object that experiences constant angular acceleration must have a constant torque applied to it. By applying a known torque to a rigid body, measuring the angular acceleration, and using the relationship $\tau = I\alpha$, the moment of inertia I can be found.

In this experiment, a torque is applied to the rotational apparatus by a string that is wrapped around the axle of the apparatus (Figs. 9-1 & 9-2).

The tension T is supplied by a hanging weight mg. The tension is found by applying Newton's second law (see Fig. 9-2).

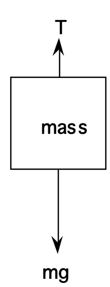

Figure 9-2 Forces on the hanging mass.

Figure 9-1

If we take upward direction as positive, and apply Newton's second law, we have

$$\sum F = T - mg = -ma, \qquad \text{Eq.1}$$

with the tension given as

$$\sum T = m(g - a) \,. \qquad \text{Eq. 2}$$

The rotational apparatus has an **original moment of inertia I_0** with no additional masses added.

When an additional **three (3) masses which are <u>assumed to be point masses</u>** are added to the apparatus, it has a new moment of inertia I_{new}. The relationship between I_o and I_{new} is given by

$$I_{new} = I_o + 3MR^2 \qquad \text{Eq. 3}$$

where 3M is the total added mass (of the point masses) and R is the distance from the center of the wheel to a point mass (i.e., R is the axis of rotation). See Figure 5.

If the masses are <u>not</u> assumed to be point masses but (rather cylinders) then using the parallel axis theorem I_{new} is given by

$$I_{new} = I_o + 3MR^2 + 3*(1/2Mr^2) \qquad \text{Eq. 4}$$

where $1/2Mr^2$ is the moment of inertia for **cylinder** rotated about its 'cylinder' axis and **r is radius of cylinder.** See figure 10.20 of text.

There is an on-line data table on lab website which also has the results format. Link is below.
https://relativity.phy.olemiss.edu/~thomas/weblab/221%20Miscellaneous%20folder/Moment_Inertia_Data%20Table_10_31_2016.pdf

Procedure

Part 1. Moment of inertia of apparatus

1. Using the vernier caliper, measure the diameter of the axle around which the string wraps. Calculate the radius r_{axle}. Make sure that no additional masses are added to the apparatus (See Figures 9-2 and 9-3 below).

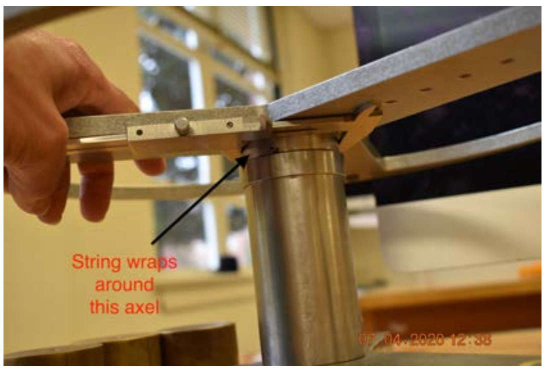

Figure 2 Axel where torque is applied

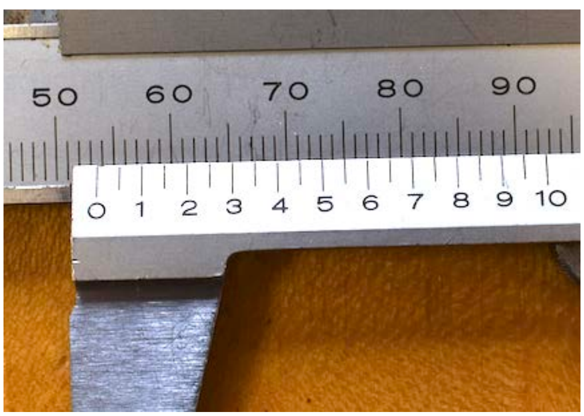

Figure 3 Diameter (2*radius) of axel from caliper

2. Wrap the string around the axle and place enough weight on the string to cause the apparatus to rotate slowly at a constant speed. The angular acceleration should be (nearly) zero. When this is the case, the sum of the torques on the body must be (nearly) zero. **FRICTION MASS WAS MEASURED TO BE 3 GRAMS.** From this data, calculate the frictional torque which is given by

$$\tau_{friction} = r_{axle} F_{friction} = r_{axle} m_{friction} g \ .$$

Steps 3 & 4 are shown on the following YouTube video.
https://youtu.be/0xMNDOaQC7c

3. Place a 50 gram mass hanger on string. Measure the distance from the bottom of the weight hanger to the floor. See figure 4 below for distance to floor measurement. Release the weight hanger, being sure not to impart an initial velocity to the wheel of the rotational apparatus.

Figure 4 Measurement of drop height

4. Use the stopwatch to time the fall (use video). Perform a total of five trials and calculate an average distance and an average time. From this information, calculate the acceleration of the mass using $distance = 1/2at^2$. Calculate the angular acceleration $\alpha = a/r$, where r is the radius of the axle that the string is wrapped around.

5. Next, calculate the tension of the string. (See the Theory section.)

6. The applied torque on the spinning wheel is provided by the tension of the string. Use the value of the tension to calculate this torque. Next, **calculate the net torque**, which is the **applied torque minus the frictional torque**.

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7. Add 50 grams to hanger **for a total of 100 grams.** Repeat steps 3-6.

8. **Plot the net torque vs. angular acceleration. Be sure to enter the origin as a data point**. Determine the moment of inertia I_o, which is the slope of the best-fit line.

Part II Additional masses added to apparatus

FRICTION MASS WAS MEASURED (WITH ADDITIONAL MASSES ADDED) TO BE 10 GRAMS.

9. Measure the distance from the center of the inertia wheel to the center of the outer set of tapped-holes**. Radius of rotation is 17.0 cm.** (*See figure below).*

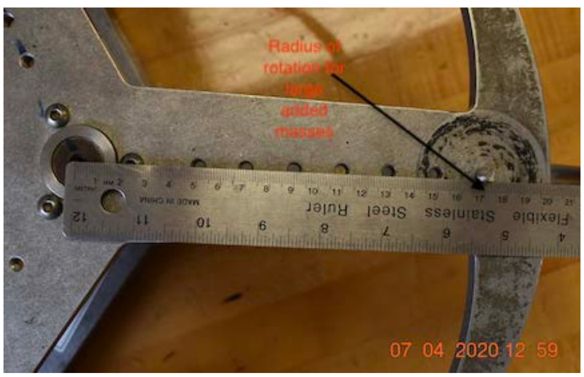

Figure 5 Radius of rotation for added masses

11. Add the total mass of the three brass masses. The mass of each one is written on the side and/or top. Attach the masses to the apparatus. Calculate the new moment of inertia, I_{new}, with these additional masses located at a distance R from the axis of rotation. **The diameter of the masses is 2.85 cm. This information will be needed to answer question 2.**

12. Repeat the steps 2 through 8 for this new moment of inertia. Plot the data and determine moment of inertia I_{new} from the slope. Calculate the percent difference between the experimental value and the calculated value. *You will do only 3 trials for steps 4 & 7.*

Questions/Conclusions

1. What is the maximum kinetic energy that the inertia device (wheel) used is given by the hanging mass (just before the mass reaches the floor). Compare this value to the gravitational energy that the hanging mass has just before you release it. *If they are not the same, explain the discrepancy.* Show all work.

2. In the theoretical determination of the moment of inertia I_{new} with the additional masses, it was assumed that the masses are points. Radius of added cylindrical masses is 2.85 cm and mass of a single added mass is 1350g. Convert to SI units.

Using the parallel-axis theorem, calculate the moment of inertia such that the diameter (radius) of the masses is taken into account. Determine the percentage difference between this and the previous value. Is it a good approximation to assume that the masses are points in this particular case?

The results of this question should be included in your discussion.

3. What is the moment of inertia of a **30 gallon steel drum if you are in sections 1 & 3 or a 20 steel gallon drum if you are in section 2 or 4)** which is rotated about the vertical axis which runs through the center of the drum sitting upright. Assume drum is a hoop. You should ignore the lid and bottom pieces of your drum.

You should search for " the size and weight of a 20 (or 30) steel gallon drum" and convert all units to SI units. Show all work and website used.